Practical Strategies for Making Math Far More Accessible to our Students

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Good Morning!

Let’s just jump right in

What is 1/10 of 450?

Convince us that 1/10 of 450 is 45.

What's the question? What's the answer?

Omaha
392 km

<u>Owner's Manual</u>
Gas Tank
Capacity:
75 liters

Average KpL:
12.8

Why did you care?

- Real and familiar context
- Could be you
- Real data, few words
- Open-ended
- Low floor
- Multiple solution paths
- Other????

Straight from the text:

Sarah has picked 2605 apples. She has 91 boxes. How many apples will Sarah put in each box if each box holds the same number of apples?

UGH!

Adapting what the text bestows

Sarah has 91 empty boxes.

- Tell us something about Sarah.

She had 2605 apples to pack into the boxes.

- What else can you say about Sarah?
- What do you think the question is?
- So what does the 91 tell us, 2605?
- About how many apples do you think would be in each box? More than 100? Less than 100? Convince us
- Can you draw a picture?
- Can you create a number sentence?
- Why did you divide and not multiply?
- So how many apples would be in each box if…

So:

Answer getting

Vs.

Explanations, alternatives, connections

In a nutshell, we know what it takes

What we strive for: LEARNING SUCCESS

The enabling conditions: ENGAGEMENT
PARTICIPATION

What it takes: TASKS QUESTIONS DISCOURSE
MISTAKES PRODUCTIVE STRUGGLE

Enhanced by: REPRESENTATIONS
ALTERNATIVES
ESTIMATES

All guided by: A CLARITY OF GOALS

Look at how we OWN the learning:

- With our tasks
- With our questions
- With our mindsets about teaching and learning

Join me in Teachers' Chat Room

- They forget
- They don't see it my way
- They approach it differently
- They don't follow directions
- They give ridiculous answers
- They don't remember the vocabulary
- They ask why do we need to learn this

THEY THEY THEY BLAME BLAME BLAME

An achievement gap or an INSTRUCTION gap?

Well…..if…..

- They forget – so we need to more deliberately review;
- They see it differently – so we need to accommodate multiple representations;
- They approach it differently – so we need to elicit, value and celebrate alternative approaches;
- They give ridiculous answers – so we need to focus on number sense and estimation;
- They don’t understand the vocabulary – so we need to build language rich classrooms;
- They ask why do we need to know this – so we need to embed the math in contexts.

Accordingly:

Some Practical, Research-Affirmed Strategies for Raising Student Achievement Through Better Instruction

Consider how we teach reading:

JANE WENT TO THE STORE.

- **Who went to the store?**
- **Where did Jane go?**
- **Why do you think Jane went to the store?**
- **Do you think it made sense for Jane to go to the store?**

Now consider mathematics:

TAKE OUT YOUR HOMEWORK.

- #1 19
- #2 37.5
- #3 185

(No why? No how do you know? No who has a different answer?)

Strategy #1

Adapt from what we know about reading

(incorporate literal, inferential, and evaluative comprehension to develop stronger neural connections)

Number from 1 to 6

1. What is 6 x 7?
2. What number is 1000 less than 18,294?
3. About how much is 32¢ and 29¢?
4. What is 1/10 of 450?
5. Draw a picture of 1 2/3
6. About how many square meters is this room? Closer to 100, 1000, 10,000 or 100,000

2nd grade:

- How much bigger is 9 than 5?
- What number is the same as 5 tens and 7 ones?
- What number is 10 less than 83?
- Draw a four-sided figure and all of its diagonals.
- About how long is this pen in centimeters?

Good morning Boys and Girls
Number from 1 to 5

1. What is the value of $\tan(\pi/4)$?
2. Sketch the graph of $(x-3)^2 + (y+2)^2 = 16$
3. What are the equations of the asymptotes of $f(x) = (x-3)/(x-2)$?
4. If $\log_2 x = -4$, what is the value of x?
5. About how much do I weight in kg?

Strategy #2

Incorporate on-going cumulative review into instruction every day.

Implementing Strategy #2

Almost no one masters something new after one or two lessons and one or two homework assignments. That is why one of the most effective strategies for fostering mastery and retention of critical skills is daily, cumulative review at the beginning of every lesson.

On the way to school:

- A term of the day
- A picture of the day
- An estimate of the day
- A skill of the day
- A graph of the day
- A word problem of the day

Great

Take a deep breath!

Oh Great

- **Ralph has 73 buttons**
- **His sister Sarah has 63 buttons**

- **How many more buttons does Ralph have than his sister?**

Alternatively

- Ralph has 73 buttons
- His sister Sarah has 63 buttons

- What's the answer?

10 136 Ralph Sarah Like buttons
They are weird 68

- What would be the question?

Tell me what you see.

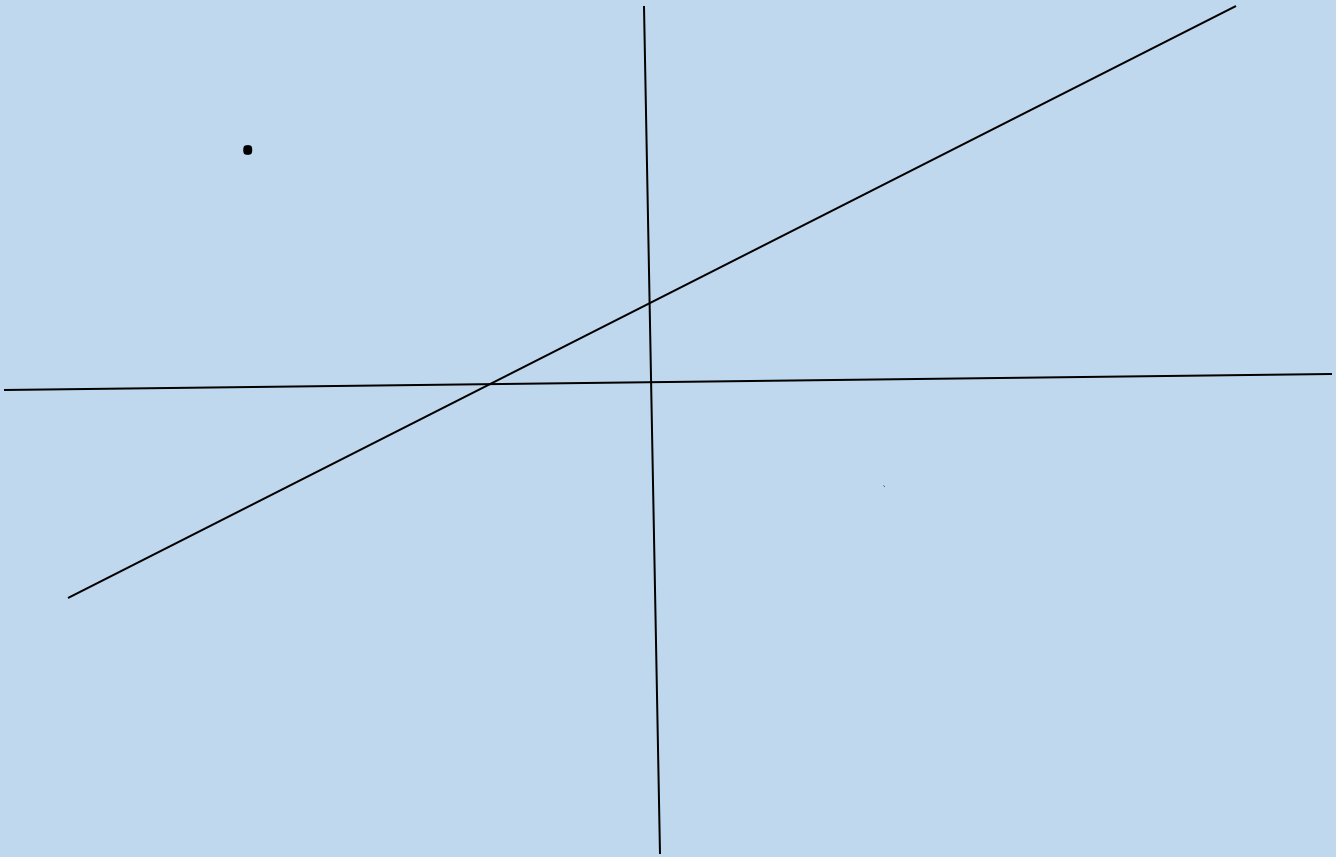
73

63

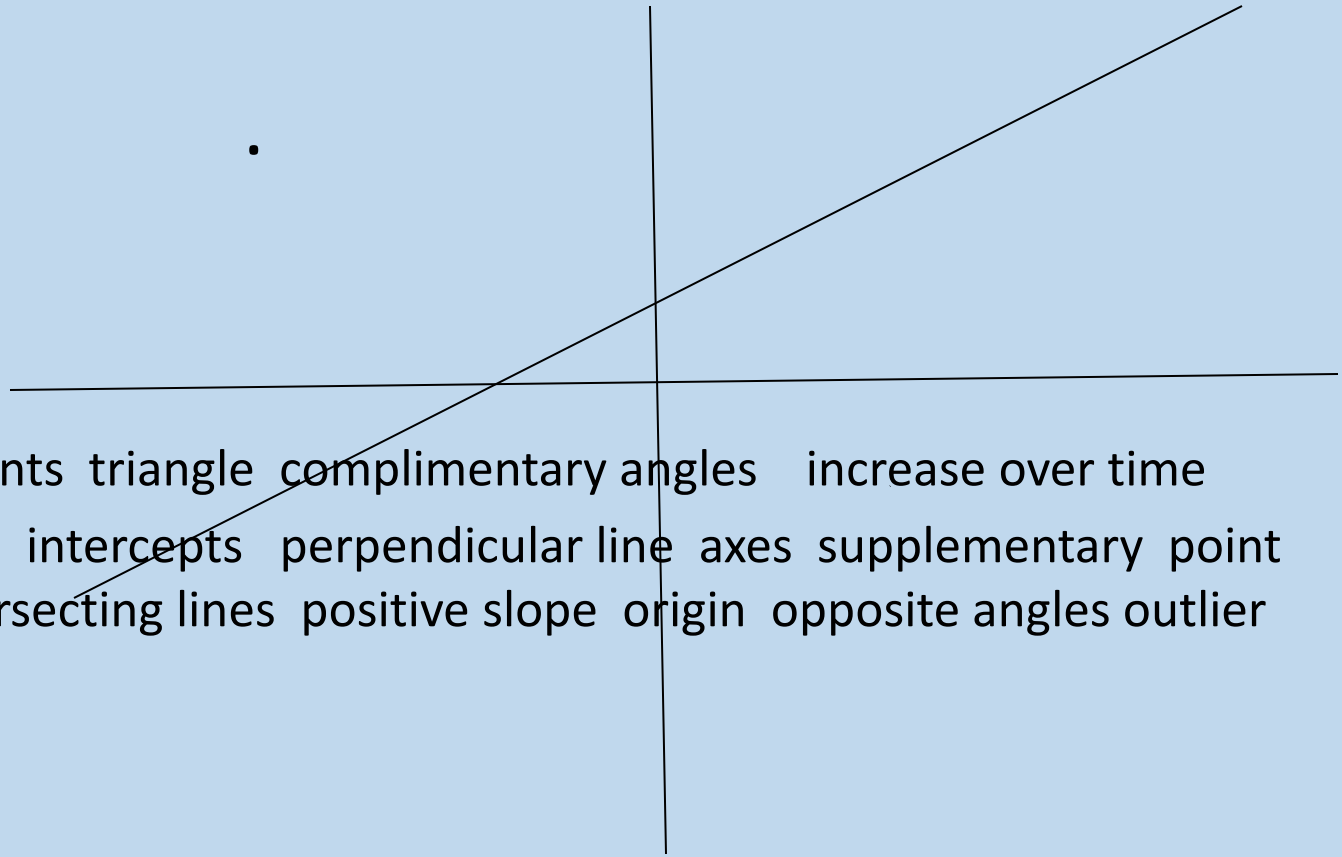
Tell me what you see.

2 1/4

Tell me what you see.



Tell me what you see.



Quadrants triangle complimentary angles increase over time

Verticle intercepts perpendicular line axes supplementary point intersecting lines positive slope origin opposite angles outlier

Strategy #3

Create a language rich classroom.

(Vocabulary, terms, answers, explanations)

Tell me what you see.

$$f(x) = x^2 + 3x - 5$$

Implementing Strategy #3

Like all languages, mathematics must be encountered orally and in writing. Like all vocabulary, mathematical terms must be used again and again in context and linked to more familiar words until they become internalized.

Sum = both
Area = covering
Perimeter = border
Circumference = a belt

Difference – bigger than
Quotient = sharing
Mg = grain of sand
Surface area = skin

Ready, set, picture…..

“three quarters”

Why does this make a difference? Consider the different ways of thinking about the same mathematics:

- **2 ½ + 1 ¾**
- **$2.50 + $1.75**
- **2 ½” + 1 ¾”**

Strategy #4

Draw pictures/
Create mental images/
Foster visualization

The power of models and representations

Siti packs her clothes into a suitcase and it weighs 29 kg.

Rahim packs his clothes into an identical suitcase and it weighs 11 kg.

Siti's clothes are three times as heavy as Rahims.

What is the mass of Rahim's clothes?

What is the mass of the suitcase?

The old (only) way:

Let S = the weight of Siti's clothes

Let R = the weight of Rahim's clothes

Let X = the weight of the suitcase

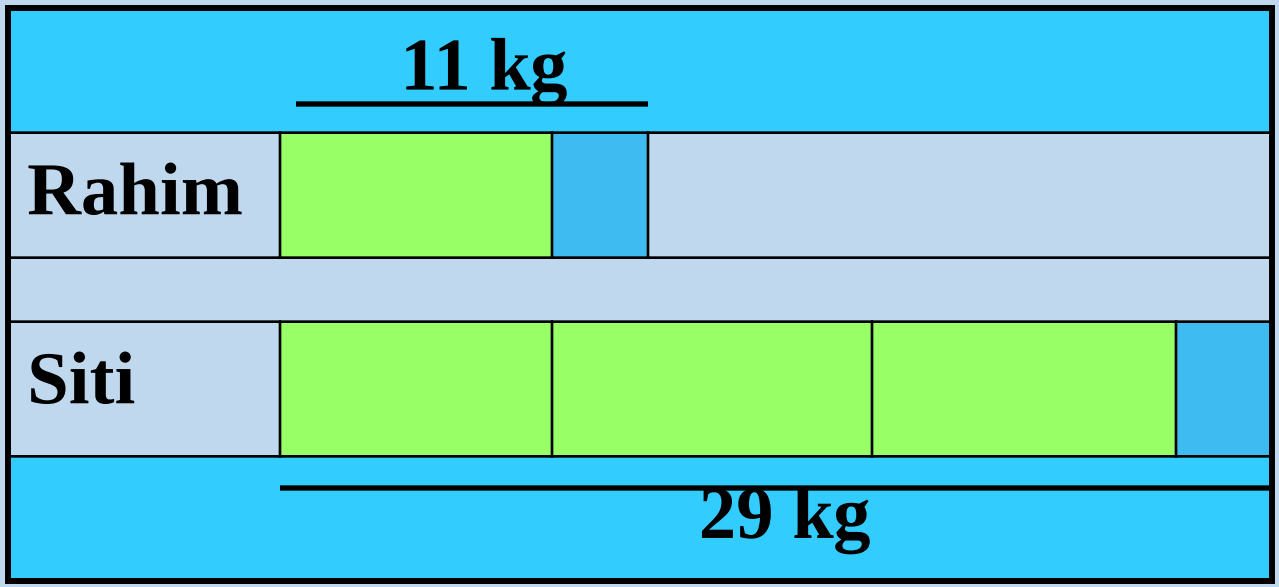
$$S = 3R \qquad S + X = 29 \qquad R + X = 11$$

so by substitution: $3R + X = 29$

and by subtraction: $2R = 18$

so $R = 9$ **and** $X = 2$

Or using a model:



Get set. Go.

What is 8 + 9?

17 Bing Bang Done!

Vs.

Convince me that 9 + 8 = 17.

Hmmmm….

8 + 9 =

17 – know it cold

10 + 7 – add 1 to 9, subtract 1 from 8

7 + 1 + 9 – decompose the 8 into 7 and 1

18 – 1 – add 10 and adjust

16 + 1 – double plus 1

20 – 3 – round up and adjust

Who's right? Does it matter?

Strategy #5

Value and elicit alternative approaches

Adding and Subtracting Integers

Remember How

$$5 + (-9)$$

Remember How

5 + (-9)

"To find the difference of two integers, subtract the absolute value of the two integers and then assign the sign of the integer with the greatest absolute value"

Understand Why

5 + (-9)

- Have $5, lost $9
- Gained 5 yards, lost 9
- 5 degrees above zero, gets 9 degrees colder
- Decompose 5 + (-5 + -4)
- Zero pairs: x x x x x o o o o o o o o o o

- On number line, start at 5 and move 9 to the left

Let's laugh at the absurdity of "the standard algorithm" and the one right way to multiply

$$\begin{array}{r} 58 \\ \times\ 47 \\ \hline \end{array}$$

3 ~~5~~

58

x 47

406

232

2726

How perfect if our goal is to continue using math to sort our students!

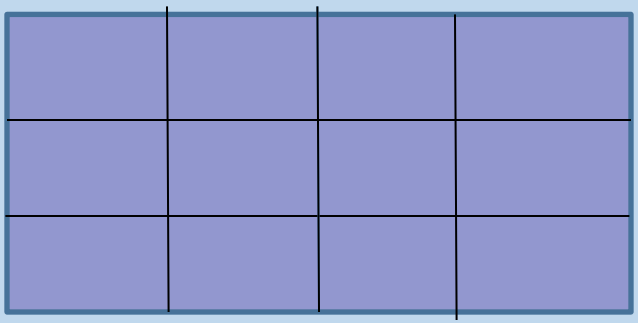
So what’s the alternative?

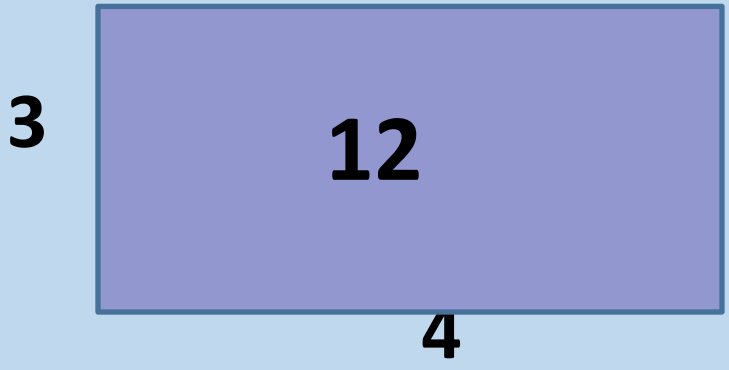
Multiplication

- What is 3 x 4? How do you know?
- What is 3 x 40? How do you know?
- What is 3 x 47? How do you know?
- What is 13 x 40? How do you know?
- What is 13 x 47? How do you know?
- What is 58 x 47? How do you know?

3 x 4

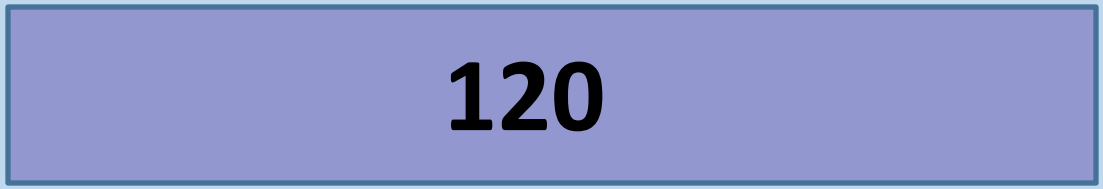
Convince me that 3 x 4 is 12.

- **4 + 4 + 4**
- **3 + 3 + 3 + 3**
- **Three threes are nine and three more for the fourth**
- 



3 x 40

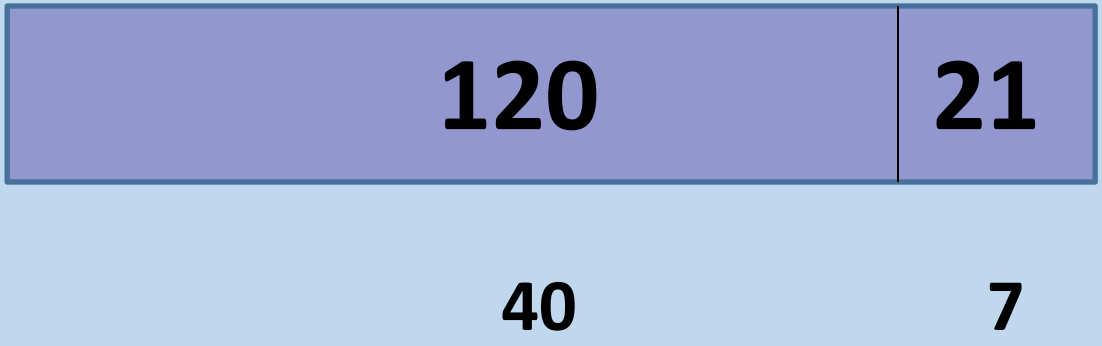
- 3 x 4 x 10 (properties)
- 40 + 40 + 40
- 12 with a 0 appended

- 3 

40

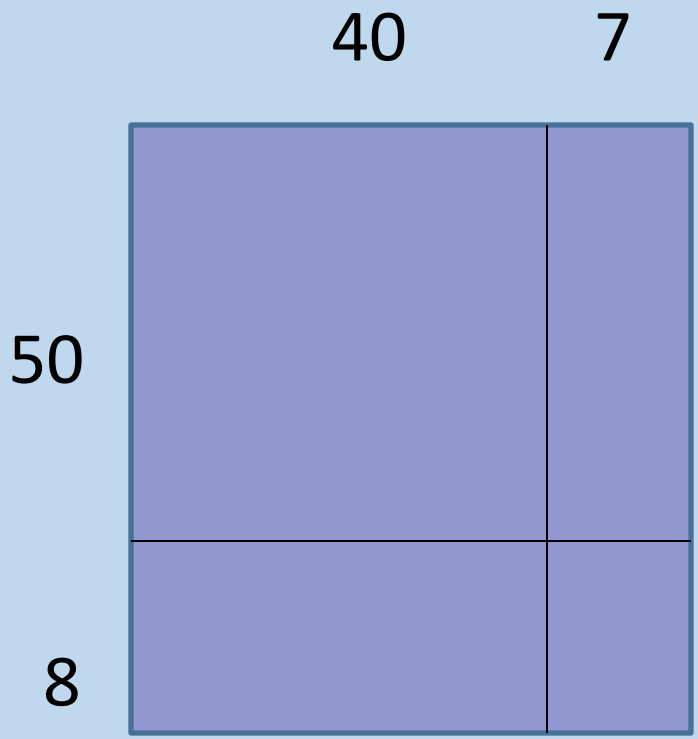
3 x 47

- 3 (40 + 7) = 3 40s + 3 7s
- 47 + 47 + 47 or 120 + 21



58 x 47

(50 + 8) (40 + 7)



58

x 47

56

350

320

2000

2726

You choose:

3 + 4 = 10 - 3 = 2 x 4 = etc.

Vs.

SALE

Pencils 3¢

Pens 4¢

Limit of 2 of each!

My Store

SALE

Pencils 3¢

Pens 4¢

Erasers 5¢

Limit of 3 of each!

SO?

Your turn

Pencils 7¢

Pens 8 ¢

Erasers 9 ¢

Limit of 10 of each.

I just spent 83¢ (no tax) in this store.
What did I purchase?

You choose:

1.59) 10

vs.

You have $10. Big Macs cost $1.59

SO?

You choose….

- **The one right way to get the one right answer that no one cares about and isn't even asked on the state tests**

vs.

- **Where am I? (the McDonalds context)**
- **Ten? Convince me.**
- **About how many? How do you know?**
- **Exactly how many? How do you know?**
- **Oops – On sale for $1.29 and I have $20.**

You choose:

Given: $F = 4(S - 65) + 10$

Find F when S = 81.

Vs.

The speeding fine in Vermont is $4 for every mile per hour over the 65 mph limit plus $10 handling fee.

Connecticut: F = 10 (S – 55) + 40

Maximum speeding fine: $350

- **Describe the fine in words**
- **At what speed does it no longer matter?**
- **At 80 mph how much better off would you be in VT than in CT?**
- **Use a graph to show this difference**

Which class do YOU want to be in?

Implementing Strategy #5

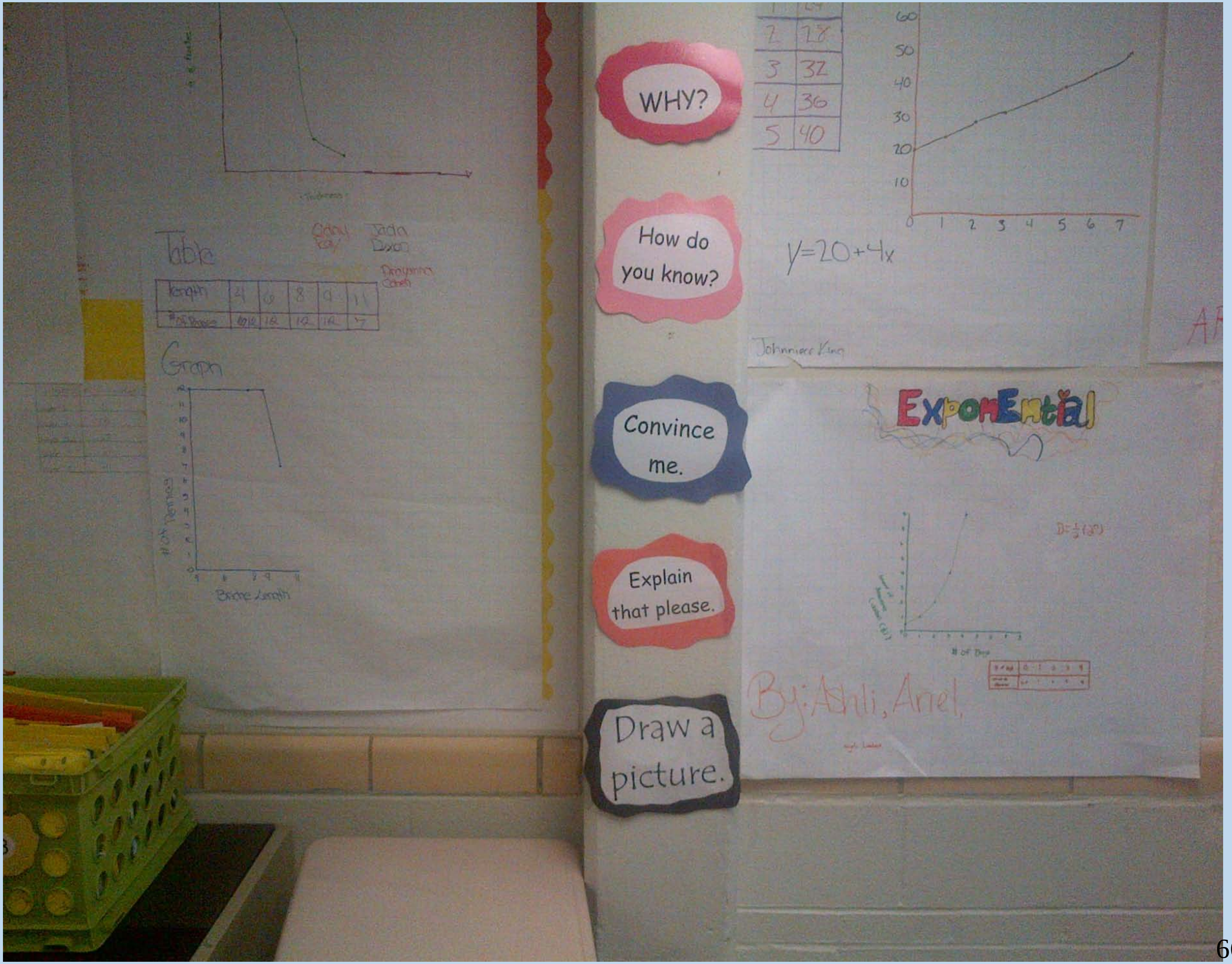
Here's the math I need to teach.

When and where do normal human beings encounter this math?

To recapitulate:

1. Incorporate on-going cumulative review
2. Parallel literal to inferential to evaluative comprehension used in reading
3. Create a language-rich classroom
4. Draw pictures/create mental images
5. Embed the math in contexts/problems

And always ask them "why?"



WHY?
How do you know?
Convince me.
Explain that please.
Draw a picture.
Table
Graph
Bridge Length
$y=20+4x$
Exponential
By: Ashli, Ariel

Classroom Posters that Drive Discourse

- Why?
- How do you know?
- Can you explain that? (and who did it differently?)
- Convince us.
- How did you picture that? (and who did it differently?)
- What do you notice?
- What do you wonder?
- How are they the same?
- How are they different?

9 Research-affirmed Practices

1. **Effective teachers of mathematics respond to most student answers with “why?”, “how do you know that?”, or “can you explain your thinking?”**
2. **Effective teachers of mathematics conduct daily cumulative review of critical and prerequisite skills and concepts at the beginning of every lesson.**
3. **Effective teachers of mathematics elicit, value, and celebrate alternative approaches to solving mathematics problems so that students are taught that mathematics is a sense-making process for understanding why and not memorizing the right procedure to get the one right answer.**

4. **Effective teachers of mathematics provide multiple representations – for example, models, diagrams, number lines, tables and graphs, as well as symbols – of all mathematical work to support the visualization of skills and concepts.**
5. **Effective teachers of mathematics create language-rich classrooms that emphasize terminology, vocabulary, explanations and solutions.**
6. **Effective teachers of mathematics take every opportunity to develop number sense by asking for, and justifying, estimates, mental calculations and equivalent forms of numbers.**

7. **Effective teachers of mathematics embed the mathematical content they are teaching in contexts to connect the mathematics to the real world.**

8. **Effective teachers of mathematics devote the last five minutes of every lesson to some form of formative assessments, for example, an exit slip, to assess the degree to which the lesson's objective was accomplished.**

9. **Effective teachers of mathematics demonstrate through the coherence of their instruction that their lessons – the tasks, the activities, the questions and the assessments – were carefully planned.**

Thank you!