# Breathing Classroom Life into the NCTM 8 Mathematics Teaching Practices

**SGIS – Zurich**

**March 10, 2017**

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# Principles to Actions:
## Ensuring Mathematical Success for All

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- **Teaching and Learning**
- **Access and Equity**
- **Curriculum**
- **Tools and Technology**
- **Assessment**
- **Professionalism**

# Principles to Actions:
## Ensuring Mathematical Success for All

**Mathematics Teaching Practices**

- **Establish mathematics goals to focus learning.**
- **Implement tasks that promote reasoning and problem solving.**
- **Use and connect mathematical representations.**
- **Facilitate meaningful mathematical discourse.**
- **Pose purposeful questions.**
- **Build procedural fluency from conceptual understanding.**
- **Support productive struggle in learning mathematics.**
- **Elicit and use evidence of student thinking.**

# Shapes

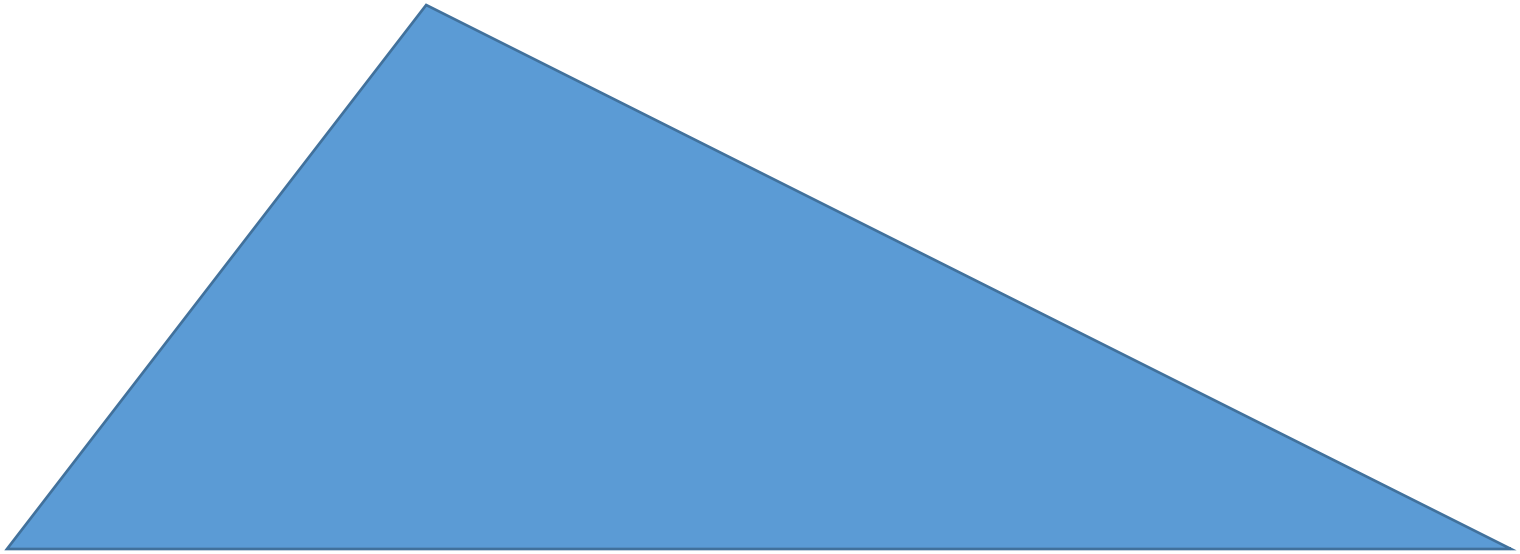
**QUESTIONS DISCOURSE PRODUCTIVE STRUGGLE**

**LEARNING GOAL: Identify Shapes**

**vs.**

**Recognize and make conjectures about the characteristics of shapes**

# What do you see?



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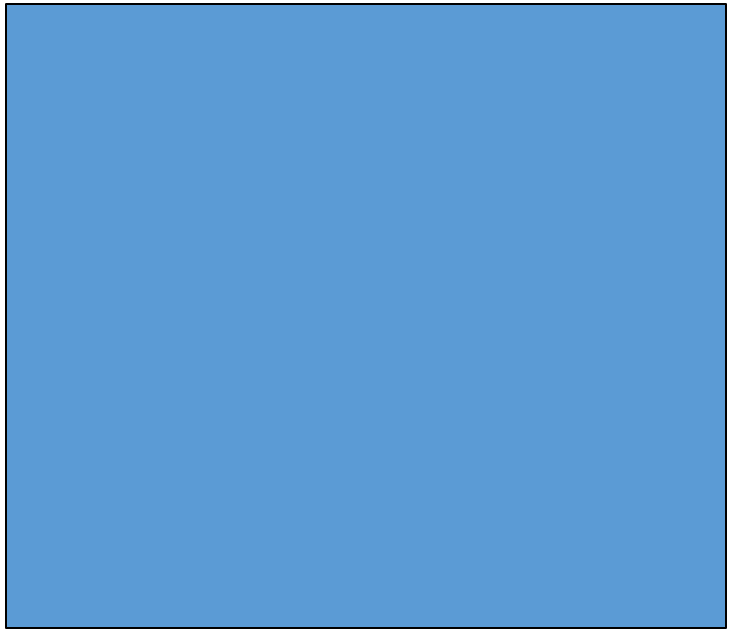
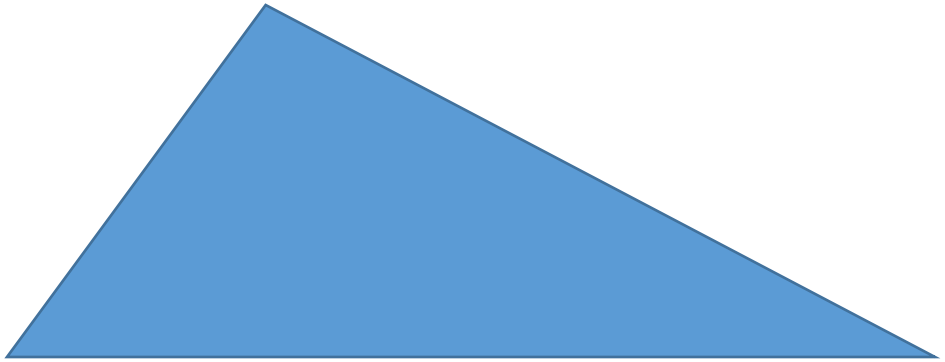


# Versus



**Identify three things you see.**

**Convince us.**

**On your white boards, A triangle is:**

**Compare to google/wikipedia**

# What is a triangle? Let's see how close we came.

- a plane figure with three straight sides and three angles.

  "an equilateral triangle"

- a thing shaped like a triangle.

  "a small triangle of grass"

- a situation involving three people or things, especially an emotional relationship involving a couple and a third person with whom one of them is involved.
- noun: **eternal triangle**; plural noun: **eternal triangles**

# Let’s Try It

**Grade 2:**

**Robert has 85 stickers.**
**Jen has 57 stickers.**

**How many more stickers does Robert have than Jen?**

**UGH!**
**The absurdity and terrible waste of one problem, one skill, one answer**
**Versus: Instead, turn and tell your partner what we can do with this.**

# Your turn

**Robert had 85 stickers.**

**What do you notice? Or tell me 3 things about Robert.**

**Jen has 57 stickers.**

**Now what do you notice?**

**What do you wonder?**

**And with partners and white boards we're off to the races!!!**

# Let’s Try It

**Grade 8/9**

**Josh and his family went to the carnival. It costs $5 to park the car plus $1.50 for each ride.**

**Josh’s family spent a total of $38. How many rides did they go on?**

**Write an equation and show your work.**

**UGH! Versus: Turn and tell your partner what we can do with this.**

# Your turn

**Welcome to the Carnival**

**Parking $5/car**

**Rides $1.50 each**

**What do you notice?**

**??**

**??**

**What do you wonder?**

**??**

**??**

# Grade 6 SA Harlem Central Tues Dec 8, 2015

- **Lesson 6 in the Expressions unit (6.EE standards)**
- **Ally and Mabubar co-teaching**
- **19 Scholars**
- **Driven by a number strings mini-lesson, a Math Workshop task and an exit ticket**
- **“Our goal for today is to “identify, create and understand equivalent expressions.”**
- **“Zayasia, can you please repeat our learning goal?”**
- **“Let’s begin with out number strings.”**

# Number strings for today's Mini Lesson

**Are they equivalent? How do you know?**

1. **4(8) = 4(3 + 5)**
2. **4(8) = 4(a + 5)**
3. **4(8) = 4(3 + b)**
4. **3x + 3y = 3(x + y)**

**Let's summarize: For each: Always, sometimes, never equivalent?**

# Math Workshop Task

**Jan normally rides her bike to and from work.**

**Her normal route is 18 miles from home to work.**

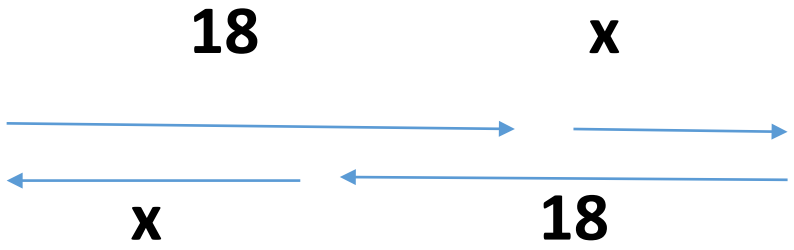
**One day she goes to a coffee shop on her way to work and on her way home.**

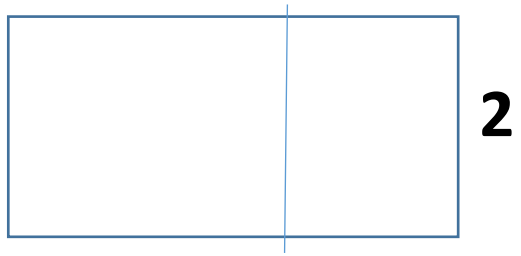
**This adds x miles to her trip each way.**

**("What do you notice?" "What's the question?")**

**Great: Write and show the distance Jan travels using a diagram or picture and two different, but equivalent, expressions.**

# Resulting in:




- **18 + x + 18 + x**
- **2(18 + x)**
- **2(x + 18)**
- **x + x + 18 + 18**
- **36 + 2x**

**“Is everyone correct?  Turn and tell your partner why?”**

**“What do the numbers and variables represent?”**

**“Which expression is simplest or easiest to use?  Why?”**

# Lesson 6 Exit Ticket

**Which of the following represent equivalent expressions?**

**Explain or show your process of determining which expressions ARE equivalent.**

**Select all that apply:**

**a. $x + x + x + x = 4x$**

**b. $15\ y + 5x = 3(5y + x)$**

**c. $6(2 + x) = 12 + 6$**

**d. $3(x + y) = 3x + y$**

# One by one, where did each of these live in this lesson?

- **Establish mathematics goals to focus learning.**
- **Implement tasks that promote reasoning and problem solving.**
- **Use and connect mathematical representations.**
- **Facilitate meaningful mathematical discourse.**
- **Pose purposeful questions.**
- **Build procedural fluency from conceptual understanding.**
- **Support productive struggle in learning mathematics.**
- **Elicit and use evidence of student thinking.**

**Establish mathematics goals to focus learning.**

# A verbatim common core standard vs.:

- **A specific learning goal in student-friendly terms;**
- **An I-can… statement;**
- **An I will understand that….statements**
- **NOT SWBAT;**
- **What is posted may not be what we put in out planning documents.**

**How do you determine lesson learning goals?**

**How do you balance skills, concepts and applications?**

**Implement tasks that promote reasoning and problem solving.**

**Find them, steal them, adapt them, revise them, reflect on them**

# Why are “tasks” so important?

- Tasks represent the meat of instruction.
- They are how we engage students  and support the development of mathematical understanding.
- They connect one’s learning goals to the actual classroom opportunity to learn.
- Exercises are opportunities to use procedures to get answers to simple problems (they have value) BUT tasks are opportunities to develop deeper and broader understanding and application of mathematics.

# In other words:

Given a set of data:

- organize the data, identify trends, ask questions, develop and justify hypotheses, summarize the data

Given a situation:

- represent it, describe it, interpret it, make predictions, plan a course of action, decide what's fair, identify contradictions or ripoffs

# In other words:

Given a claim:

- evaluate it

Given a set of constraints:

- satisfy them, find an optimum solution

Given a model:

- explain it, debug it, refine it, generalize it, evaluate it, apply it

***Notice how this aligns with the CCSSM Standards for Mathematical Practice.***

# How do we find good tasks?

# Great On-line Math Resources

Learn Zillion: www.learnzillion.com

Inside Mathematics: www.insidemathematics.org

Illustrative Mathematics: www.illustrativemathematics.org

NCTM Illuminations: http://illuminations.nctm.org

Balanced Assessment: http://balancedassessment.concord.org

Mathalicious: http://www.mathalicious.com

Dan Meyer's three act lessons: https://docs.google.com/spreadsheet/ccc?key=0AjIqyKM9d7ZYdEhtR3BJMmdBWnM2YWxWYVM1UWowTEE

Thinking blocks: http://www.thinkingblocks.com

Math Assessment Project http://map.mathshell.org/materials/index.php

Yummy Math: www.yummymath.com

**See www.steveleinwand.com**

(publications; great online resources)

# And best of all:

Emergent Math: http://emergentmath.com/my-problem-based-curriculum-maps

# How do we decide if a task is good?

**Accessible** – clear and has multiple entry points

**Fair** – all students are able to complete it

**Reasonable** – not too complex, familiar context, not too arcane

**Aligned** – matches standards and current learning goals

**Comprehensive** – integrates key understandings
– big enough bang for the time

**Engaging** – uses graphics
– has an intriguing or familiar context

**Divergent** – provides multiple pathways to solve

# When might we use such tasks?

At first:

Once or twice a week.

Definitely Friday and often on Monday as well.

That is, such tasks can launch and give students an understanding of WHY they are learning this math.

And they should culminate chunks of instruction and give students and teachers a clear sense that what is important has been learned.

# How do we engage ALL students in persevering through the tasks?

- Gradual reveal (acclimating/familiarizing)
- Warm-up questions
- “What are you being asked here?”
- “What do you notice?”
- Take a guess
- Draw a picture
- Make a movie

- All BEFORE “solve”

**Use and connect mathematical representations.**

# Where would we be without models?

Jacob bought 280 blue and red paper cups. He used $\frac{1}{3}$ of the blue ones and $\frac{1}{2}$ of the red ones at a party. If he had an equal number of blue cups and red cups left over, how many cups did he use altogether?

# The power of models and representations

**Siti packs her clothes into a suitcase and it weighs 29 kg.**

**Rahim packs his clothes into an identical suitcase and it weighs 11 kg.**

**Siti's clothes are three times as heavy as Rahims.**

**What is the mass of Rahim's clothes?**

**What is the mass of the suitcase?**

# The old (only) way:

**Let S = the weight of Siti's clothes**

**Let R = the weight of Rahim's clothes**

**Let X = the weight of the suitcase**

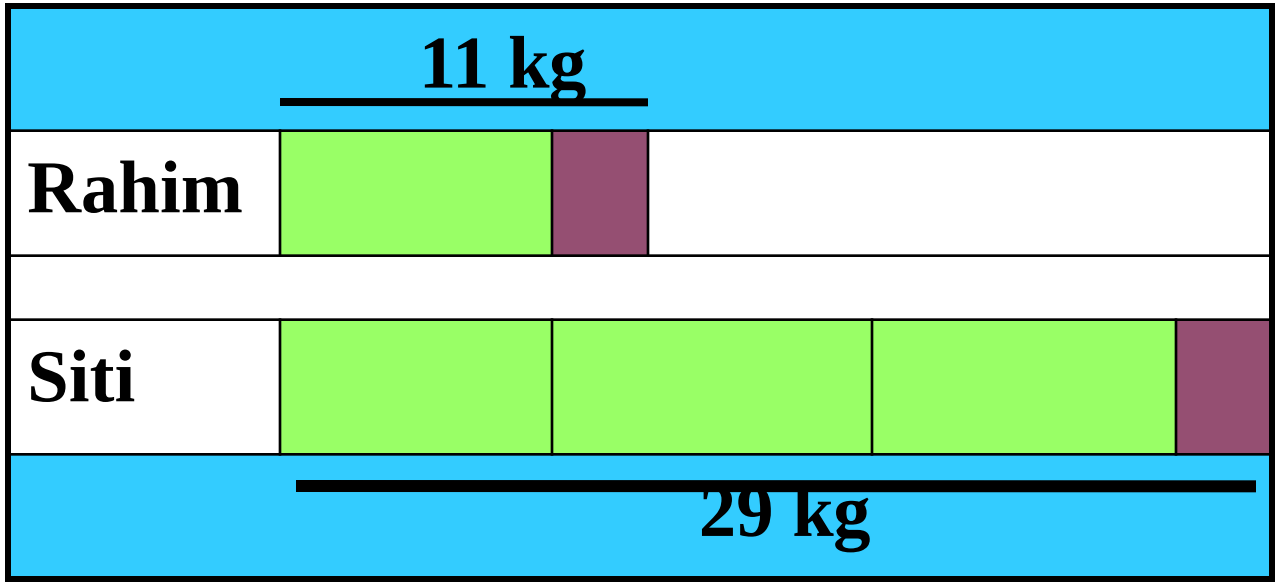
**S = 3R  S + X = 29  R + X = 11**

**so by substitution: 3R + X = 29**

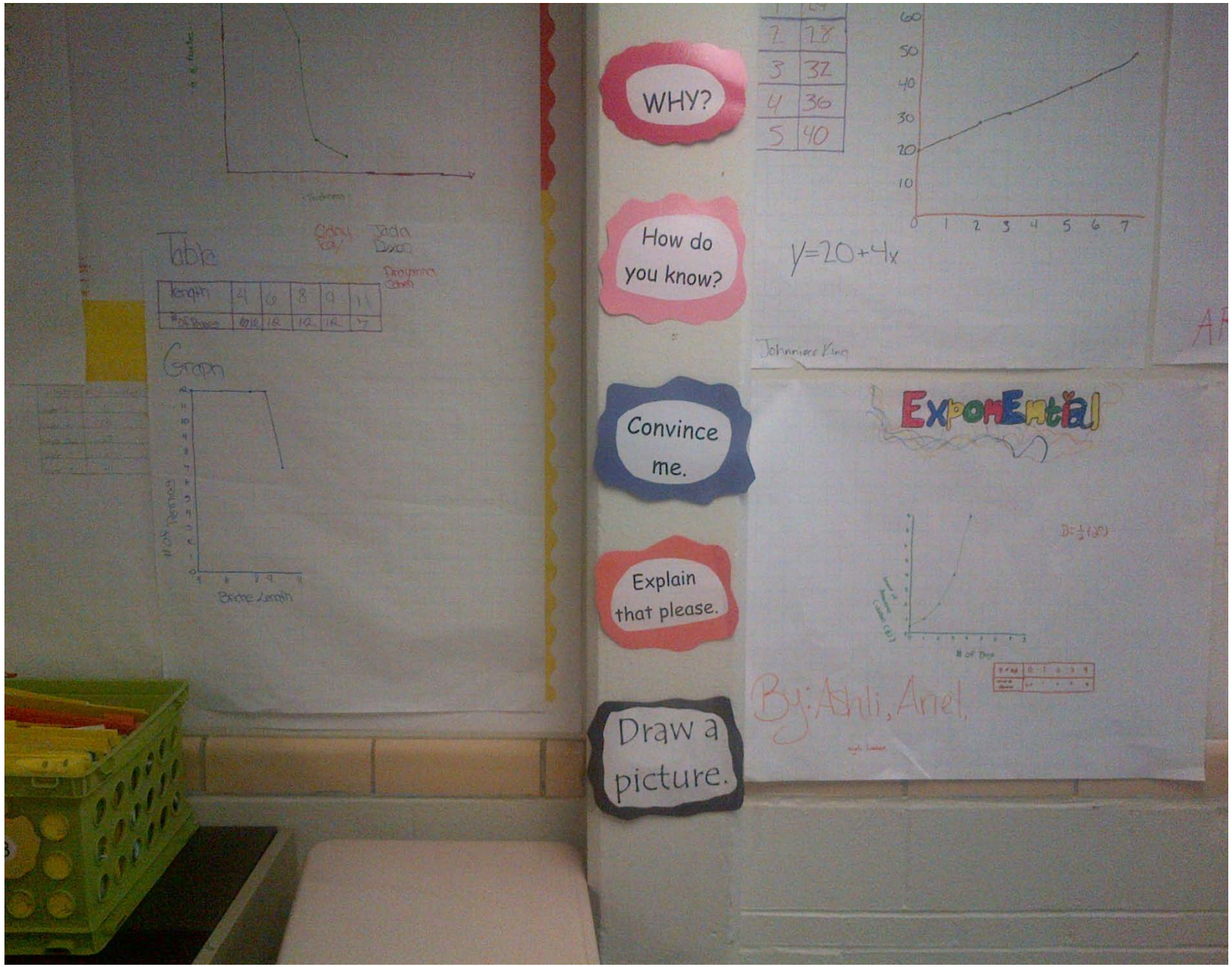
**and by subtraction: 2R = 18**

**so R = 9 and X = 2**

# Or using a model:



**Facilitate meaningful mathematical discourse.**


WHY?
How do you know?
Convince me.
Explain that please.
Draw a picture.
Table
Graph
y=20+4x
ExponEntial
By: Ashli, Ariel

# Classroom Posters that Drive Discourse

- **Why?**
- **How do you know?**
- **Can you explain that? (and who did it differently?)**
- **Convince us.**
- **How did you picture that? (and who did it differently?)**
- **What do you notice?**
- **What do you wonder?**
- **How are they the same?**
- **How are they different?**

**Pose purposeful questions.**

# What is 18 ÷ 3???

**Awesome:**

- **Write that as multiplication.**
- **Show your answer as equal groups.**
- **Show your answer as an array or area model.**
- **Show where your answer fits into a multiplication table.**
- **Write a word problem that requires you to divide 18 by 3.**

**Awesome: What about 19 ÷ 3?**

# How much is: 8/5 ÷ 0.7

- **More than 2 or Less than 2? How do you know?**
- **Exactly how much?**
- **Turn and tell your partner what you think the answer is?**
- **(Collect answers)**
- **Can they all be correct?**
- **Turn and convince your partner which answer is correct?**
- **Who can explain?**
- **Who did it differently?**
- **What did you just learn?**

# Tweaking their brains:

1. Write a story problem that requires a normal human being to find 8/5 divided by 0.7.
2. Create a model – convince someone that your model works
3. If you did this with fractions, do this with decimals, vice versa
4. Explain orally and in writing
5. 7/6 divided by 0.15
6. Our original problem had an answer a little more than 2.

    change the 8/5 so that the answer is less than 1

    change the 0.7 so that the answer is less than 1

So:

**Answer getting**

**Vs.**

**Explanations, alternatives, connections**

**And look at the power of our questions!**

**Build procedural fluency from conceptual understanding.**

# Convince us that 7 x 6 is 42.

**Support productive struggle in learning mathematics.**

# Ready!

**At Central Middle School, there are 3 scholars in the Science Club for every 8 scholars in the Math Club.**

**If there are a total of 45 scholars in one or both of these clubs, how many scholars are in both clubs?**

- Math 8x  Science 3x  Total: 11 x  11 x 5 = 55, so 10 in both

  Ergo:

| Math | Science | (Both) | Total (45) |
|---|---|---|---|
| 8 | 3 | | 11? |
| 16 | 6 | | 22? |
| 24 | 9 | | 33? |
| 32 | 12 | | 44? |
| 40 | 15 | 10 | 55 |

**Elicit and use evidence of student thinking.**

**How difficult is it to appoint two “5-minutes-left time monitors”?**

**With 5 minutes to go:**

- **turn and tell your partner what you learned today**
- **quick “tweets” to share**
- **Ok, let’s see.  Turn your exit slip over and complete the task**

# So let’s put it all together again:

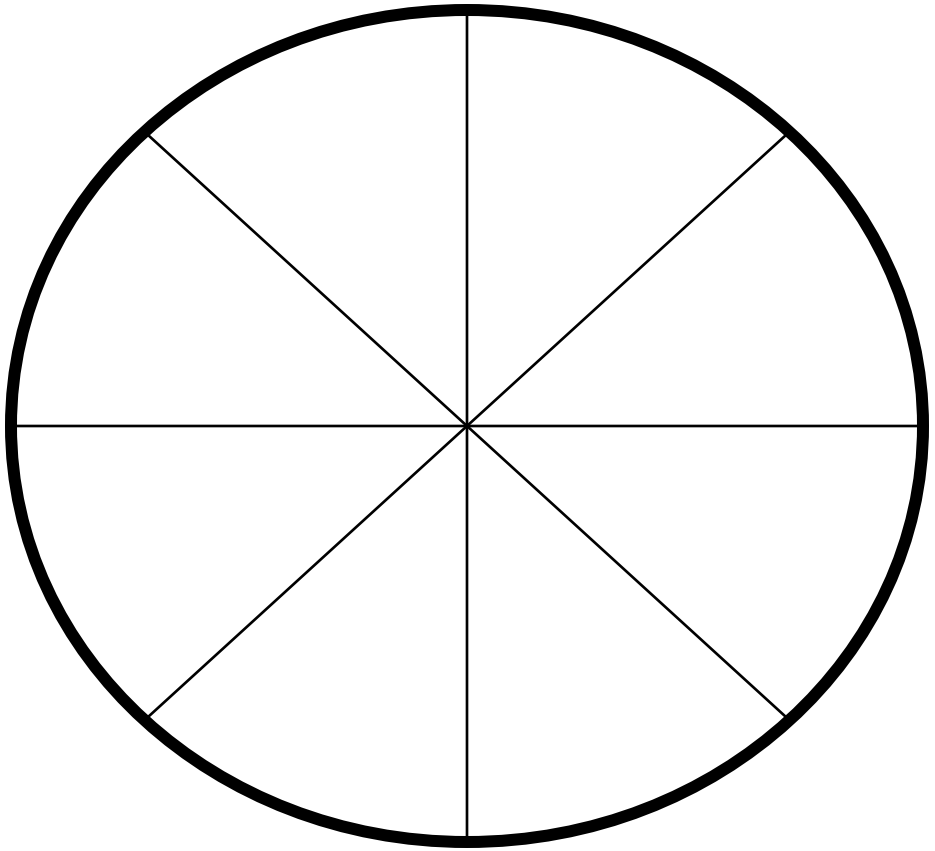
Grade 3

**Good morning third graders!**

**We've been studying fractions for a while.**

**Let's review with a focus on common misunderstandings.**

# In your groups, write down 5 things you see



# Tim and Sari

**Tim eats 1/2 of a pizza.**

**Sari eats 2/4 of a pizza that is the same size.**

**Who eats more pizza?  How do you know? Can you use a drawing to convince us?**

# Tim and Sari 2

**Tim eats 1/4 of a pizza.**

**Sari eats 1/5 of a pizza that is the same size.**

**Who eats more pizza?  How do you know? Can you use a drawing to convince us?**

# Tim and Sari 3

**Tim and Sari each eat ½ of a pizza.**

**Tim says they both ate the same amount.**

**Sari says she ate more than Tim.**

**Show with pizza drawings for Tim and Sari that both can be right!!!**

# Tim and Sari 4

**Tim eats 3/4 of a pizza.  Show this.**

**Sari's pizza has 8 slices and is the same size of Tim's.  Show this.**

**How many slices does Sari need to eat so that she and Tim eat the same amount?**

**Show with pizza drawings how you know.**

# So, given all of this, I've been collaboratively developing 6- to 8-slide lesson guides

- **Objective or Lesson Goal slide student friendly with underlining to focus on understanding of vocabulary and goal**
- **Warm-up slide(s) – to get the blood flowing and focus on key prerequisite understandings**
- **Tasks with Questions and all necessary graphics – so others can use without additional support**
- **WDYLT**
- **Exit Slip**

# Design Questions

1. **What are you trying to accomplish in this lesson? OK frame your opening lesson goal slide.**
2. **What would students be able to do to convince us the lesson worked. OK build your exit ticket slide.**
3. **What tasks, examples, problems, activities will students be engaged in to maximize the chance they will successfully meet the lesson goals and complete the exit ticket? OK craft your core lesson slides.**
4. **What pre-requisite understandings are likely to be essential for the tasks, examples, etc. to work? OK craft your warm-up slides.**

# Good Morning 3BC

**Math for Monday, October 10, 2016**

**Today's Math Goal:**

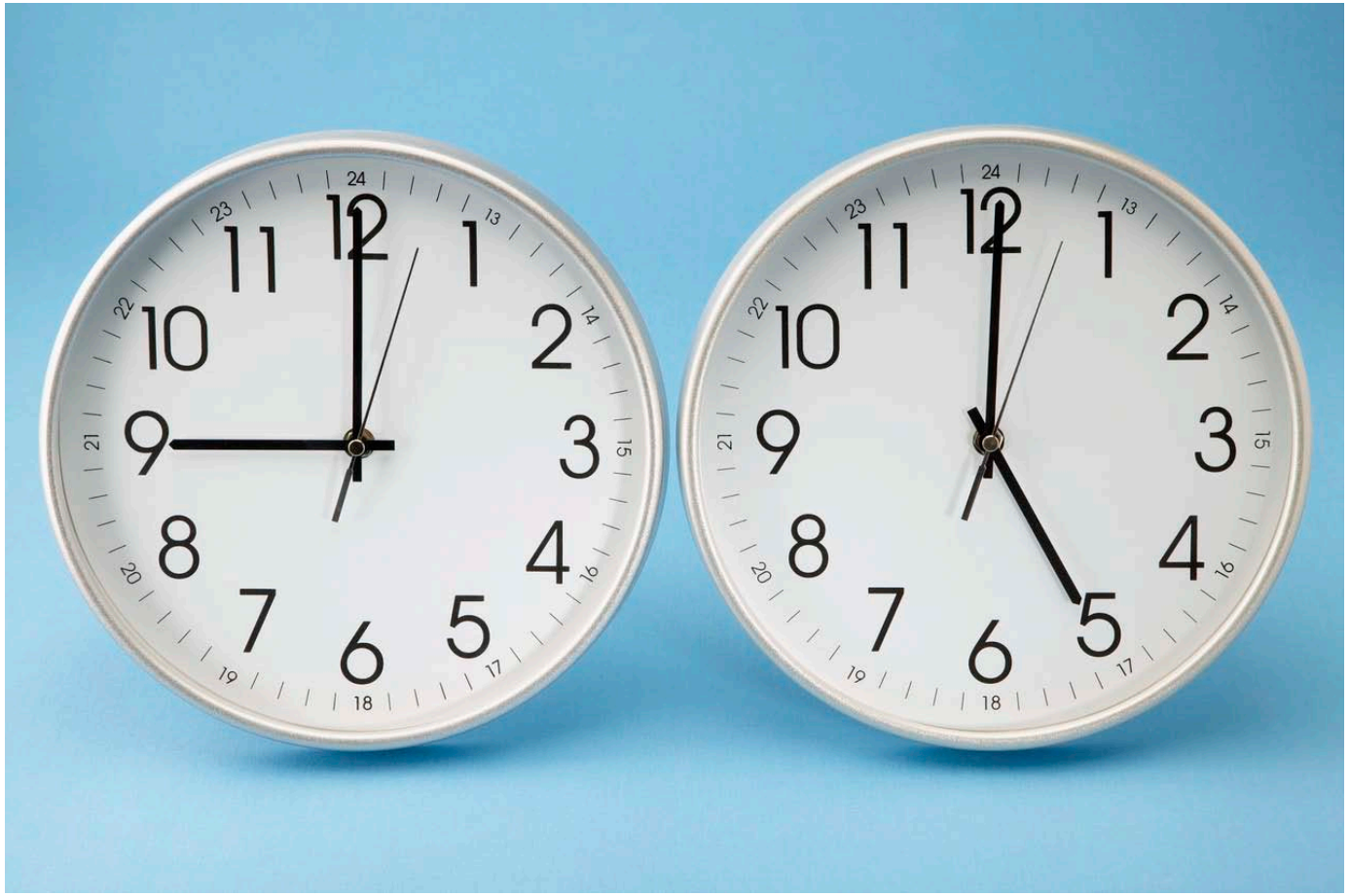
**To use a number line and skip counting to tell time on a clock.**

# Warm up

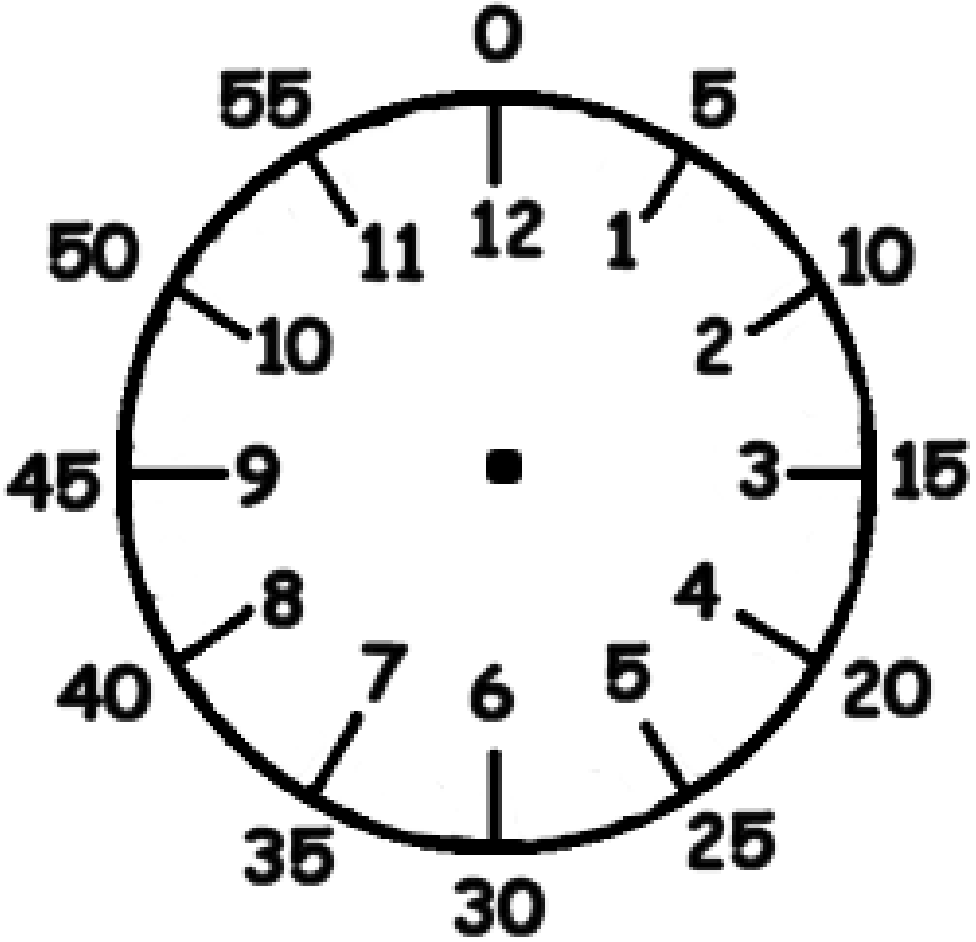
**Let’s skip count:**

- **By 2’s to 20**
- **By 7’s to 35**
- **By 8’s to 40**

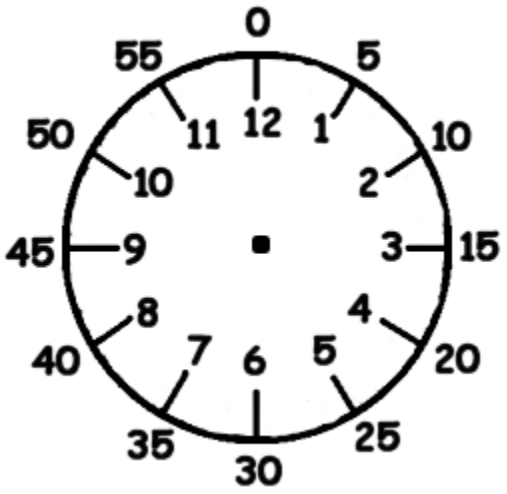
# Tell your partner three things you notice.



# Now what do you see?



# Problem to solve

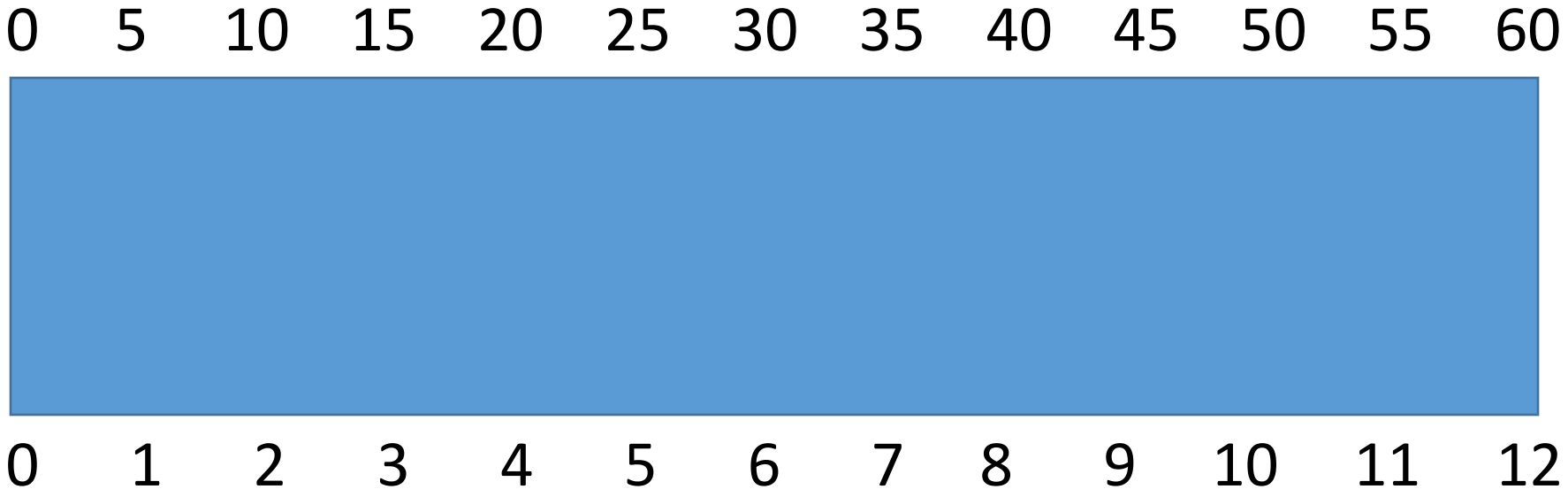



**Christine has 12 math problems.**

**It takes here 5 minutes to complete each problem.**

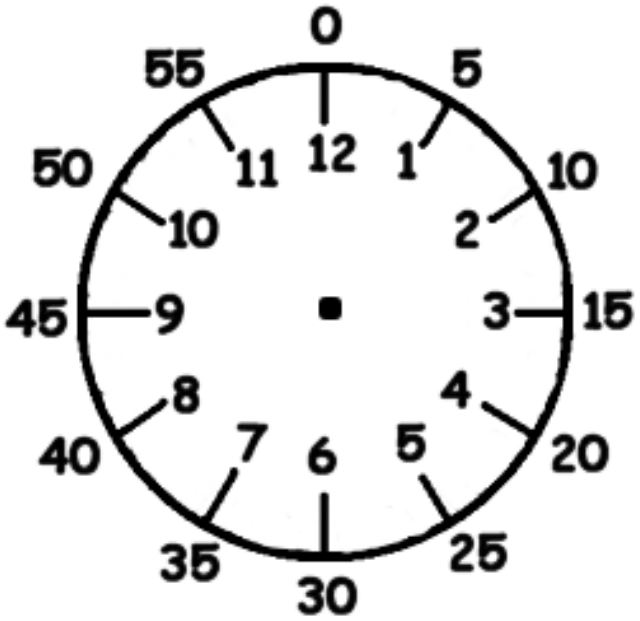
**How long will it take her to do all 12 problems?**

# Now what do you notice?




**How many problems did Christine do? How long did it take her?**

# Solve problems with time




1. It’s 20 after 10. Where does the minute hand point?
2. You start your homework at 4:30. You finish at 4:55. How long did you work?
3. It was just 8:00. The minute hand is now on the 3. What time is it?

# What did you learn today?

# Exit Ticket

**It is 7:40.**

1. **Show this on a clock face.**
2. **When will it be 8:00?**
3. **What time was it 25 minutes ago?**
4. **You start your homework 7:40. Each problem takes 5 minutes. At what time will you finish 9 problems.**

# Good Morning 5EP

**Math for Monday, October 10, 2016**

**Today's Math Goal:**

**To learn about use mental math strategies for multiplying by multiples of ten.**

# Warm Up

**Solve and explain.  Ready:**

1. **3 x 7**
2. **3 x 70**
3. **3 x 700**
4. **30 x 7**
5. **Why is 30 x 7 the same as 3 x 70?**

# More warm up

**What is the value of 10 tens?**

**What is the value of 10 hundreds?**

**What is the value of 100 hundreds?**

**What is the value of 100 thousands?**

Multiplying by multiples of 10.
Let’s see if this makes sense.

**Solve and explain:**

**4 x 5**

**40 x 5**

**40 x 50**

**40 x 500**

**400 x 500**

# True or False?  Convince Us.

**1. 2 tens x 3 tens = 60 tens**

**2. 44 x 20 x 10 = 440 x 20**

**3. 86 ones x 90 hundreds = 86 ones x 900 tens**

**Would your rather have 50 hundreds or 5000 tens?**

# Niagara Falls

**Water flows over Niagara Falls at a rate of 600,000 gallons per second.**

**How many gallons is this per minute? _____**

# What did you learn today?

# Exit Ticket

**How much money was made?**

| | Adults | Children |
|---|---|---|
| **Number** | **45** | **600** |
| **Cost** | **$20** | **$15** |

**Try to do this mentally.**

# Good Morning 2HE

**Math for Monday, October 10, 2016**
**Today's Math Goal:**
**To learn about and measure with a ruler.**

**(we'll need white boards, rulers, and cubes)**

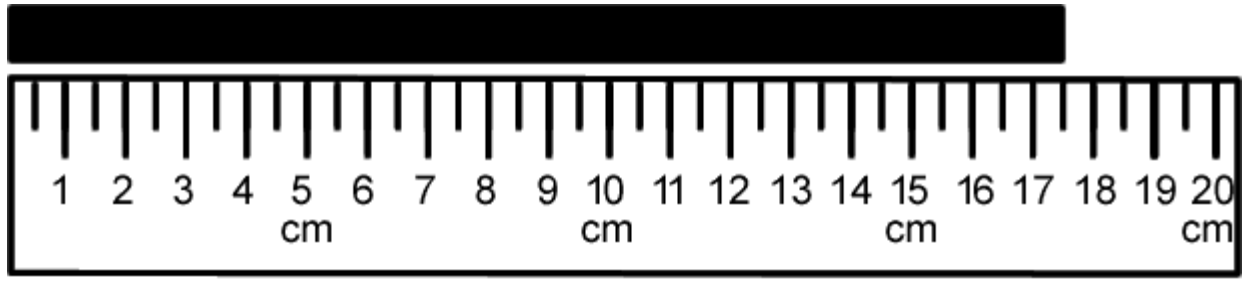
# Warm up 1

**Look at this book.**

**How big is this book?**

# Warm up 2

**Look at this ruler.**




**With your partner, identify 4 things you notice about your ruler.**

# Let’s do some measuring

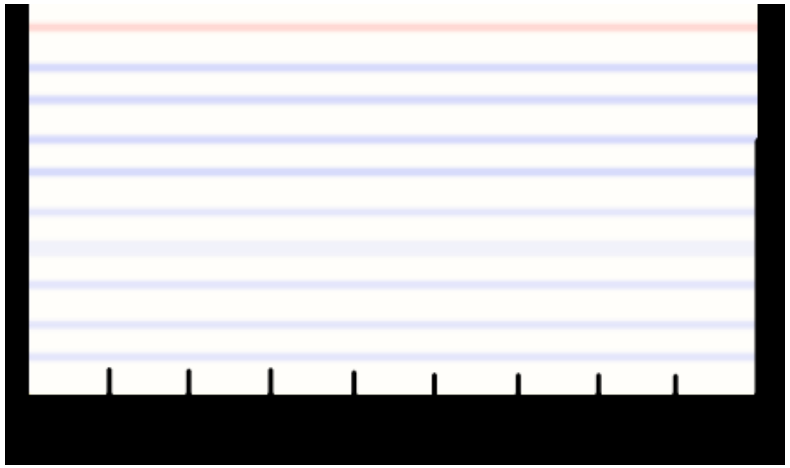
**Here is a line.  How long is the line?**

1. **Can you estimate? Turn and tell your partner how long you think the line is.**
2. **How many hands?**
3. **How many markers?**
4. **How many centimeters?**
5. **How do we measure length?**

# How long is my arm?

- **Write down your estimate.**
- **What unit did you use?**
- **What unit should we use?**
- **How do we use our ruler?**
- **How long is my arm?**

Matt measured this index card using a ruler. He thinks the index card is 10 centimeters long.




Is Matt correct? Explain why or why not.

**Why do you think we use rulers to measure length?**

# What did you learn today?

# Exit Ticket

**Draw a line on your white board that is about the length of your hand.**

1. **Estimate the length in cubes and in centimeters.**
2. **Measure the length in cubes**
3. **Measure the length in centimeters?**
4. **How long is your hand?**

# Good Morning 4NI

**Math for Monday, October 10, 2016**

**Today's Math Goal:**

**To learn about and apply metric measurement conversions.**

# Warm up 1

**Look at this book.**

**How big is this book?**

# Warm up 2

**What does it mean to MEASURE something?**

# Using our rulers

**Find two ways to tell me how long your marker is.**

**How can the marker be two DIFFERENT lengths?**

**What did we discover about the length in cm and mm?**

# Find a different object.

- **How long is it in cm?**
- **How long is it in mm?**
- **Did you really need to measure twice?**
- **If you know the length in cm, how do you find the length in mm?**
- **If you know the length in mm, how do you find the length in cm?**

# Ready?

**My ribbon is 8 centimeters long.**

**Your ribbon is 30 millimeters long.**

**How much ribbon do we have together??**

**Who answered in cm?  Who answered in mm?**

# Let’s see if this makes sense.

**My brother is 1 meter tall.**

**I am 70 centimeters tall.**

**My baby sister is 500 millimeters tall.**

**Who is tallest?**

**Who is shortest?**

**What is our total height in cm?**

**What is our total height in mm?**

# What did you learn today?

# Exit Ticket

**A piece of yellow ribbon is 400 mm long.**

**A piece of red ribbon is 30 centimeters long.**

**A piece of blue ribbon is 2 meters long.**

1. **Which color ribbon is longest?  Shortest?**
2. **What is the total length of the 3 pieces of ribbons?**
3. **What is another way to write the length of the 3 pieces of ribbon?**

# Good Morning 1EG

**Math for Monday, October 10, 2016**

**Today's Math Goal:**

**To use models to solve subtraction problems.**

# Ready to Model?

**Maggie has 5 apples.**

**Tell me about Maggie.**

**On your white boards, use your cubes to show me Maggie's apples**

**Mr. Steve came along and took two of Maggie's apples.**

**Can you show me what Mr. Steve did.**

**Do you now have more or less apples?**

**Show me how many apples Maggie had left?**

**So we have shown that: 5 – 2 = 3.**

# Do you want to try another one with cubes?

**Luke has 8 pencils.  Show this.**

**Luke gave 5 of his pencils to Mr. Steve.**

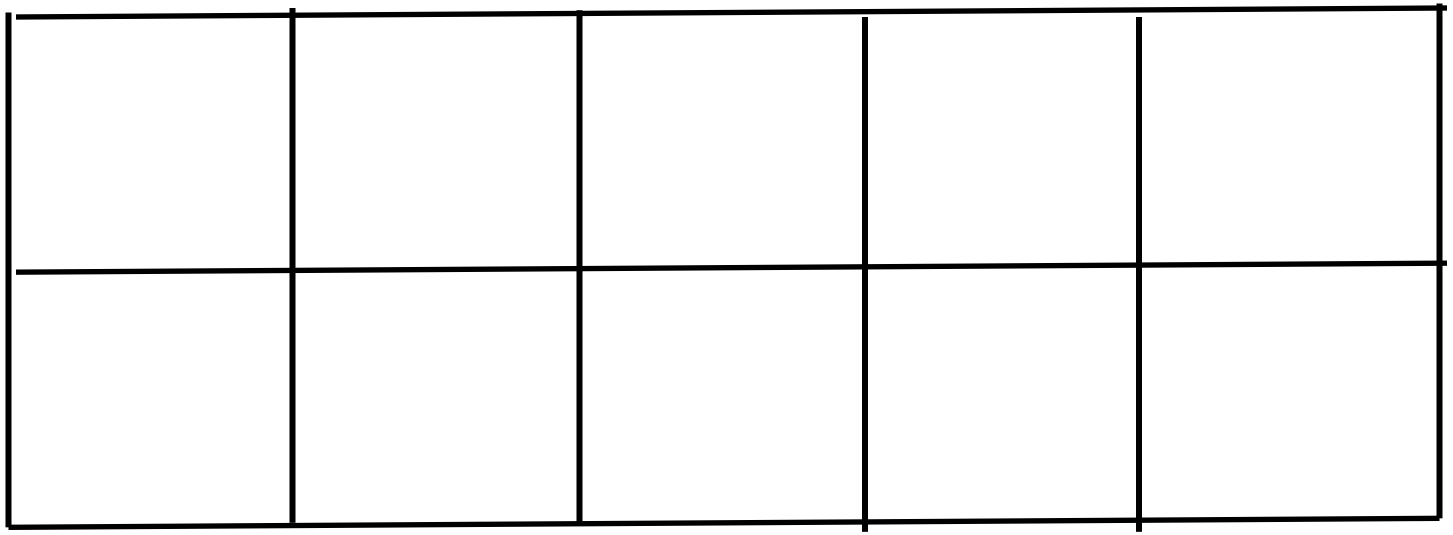
**How many pencils does Luke have left?**

**What number sentence can we use to show this?**

# Let’s try to act our next problem.

- **Su Mei caught 10 fish today.**
- **Can we show these 10 fish using our class?**
- **Are you ready?**
- **She caught 7 fish in the morning.**
- **What’s the question?**
- **How many fish did she catch in the afternoon.**
- **Can we show this?**
- **10 – 7 = <u>3</u>  7 + <u>3</u> = 10**

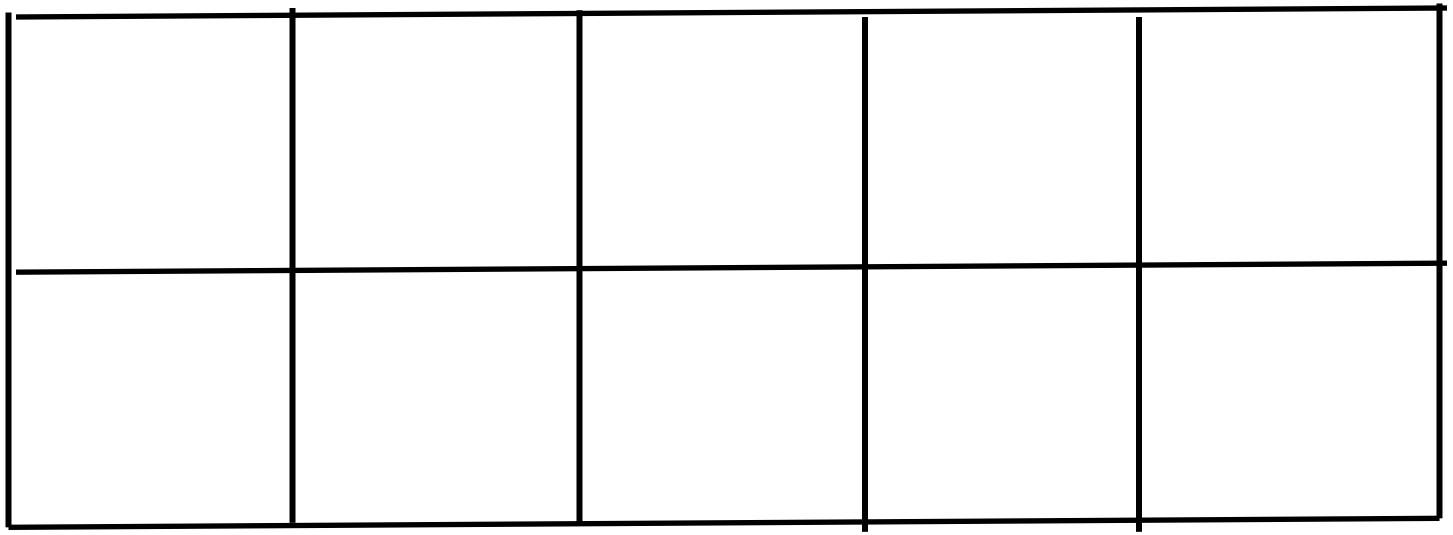
# Are you ready for a new model and bigger numbers?



**Can you draw this on your white boards?**

# Can we use the Tens Frame to model this problem?

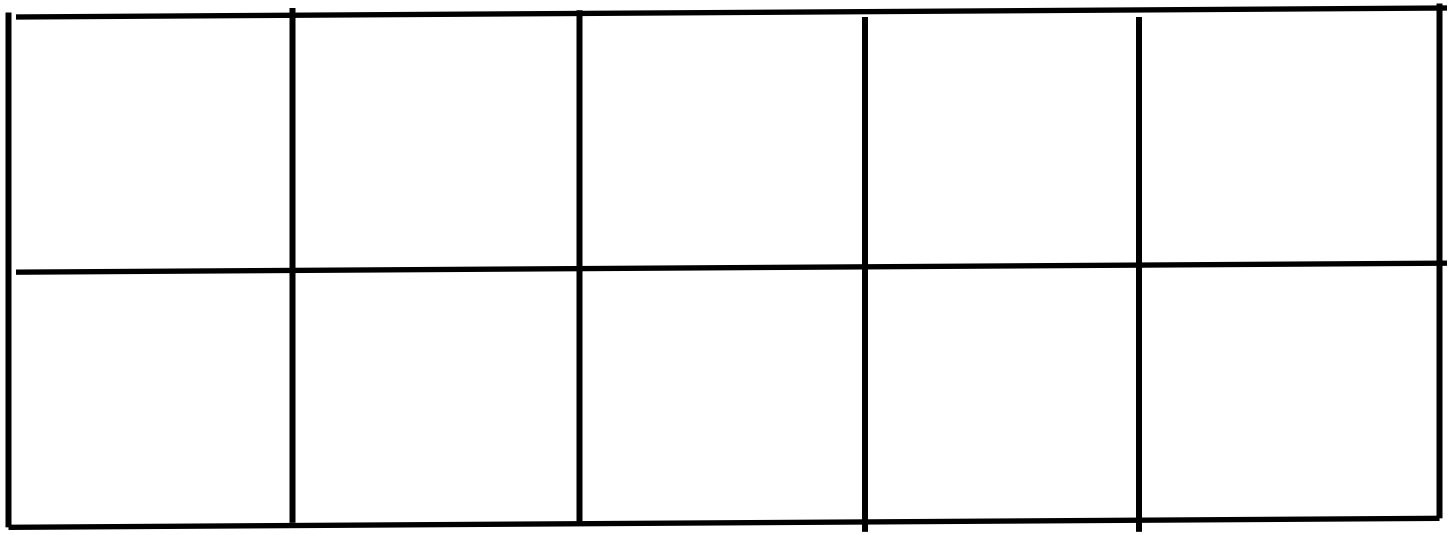
**Su Mei caught 10 fish today.  She caught 7 in the morning.**



# One more:

**Pam and Kate are playing with 7 trucks. Pam is playing with 2 of the trucks. How many trucks is Kate playing with.**

**LET'S MODEL.**



**What number sentences can we write?**

**Who can explain their thinking?**

# What did you learn today?

# Grade 7 Geometry

7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

# Learning Goals

- Estimation
- Solve problems of volume of triangular prisms

What do you notice?

What do you wonder?


**About how much chocolate is in the box?**

# About how much chocolate is in the box?

Too high

Too low

# Now what?


Debrief Questions:

- Was your initial estimate too high / too low? Why?
- What shape did you use to model the problem?
- How did you account for the gaps in the chocolate bar?

# Exit Ticket

What is the approximate volume of the box?

# Good Morning Geometry!

# Dilation: A Special Case of Similarity

**Monday, October 17, 2016**

# Dilation, to Dilate something

- **What does it mean to dilate?**
- **What does NOT change when something is dilated?**
- **What DOES change when something is dilated?**
- **How is dilation related to similarity?**

# Scale Drawings

**What are scale drawings?**

**Are scale drawings dilations? Why? Why not?**

**What is the prominent feature of a scale drawing?**

**Yes, so what is the “scale” in a scale drawing?**

# So what do you see?

# What's changed?

How has Teddy grown?

Can we find the scale factor?

# With a partner:

**Draw a square.  Label it Figure 1.**

**Dilate your square by a scale factor of 0.5 and label it Figure 2.**

**Dilate your square by a scale factor of 2 and label it Figure 3.**

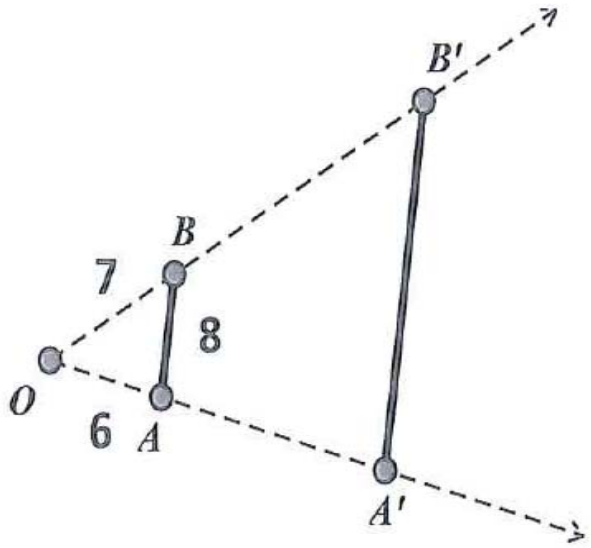
**Compare the changes in side lengths and areas of the three squares.**

# The amazing thing about dilations:

**Let’s line up our Teddy bears again.**

**Let’s line up our squares again.**

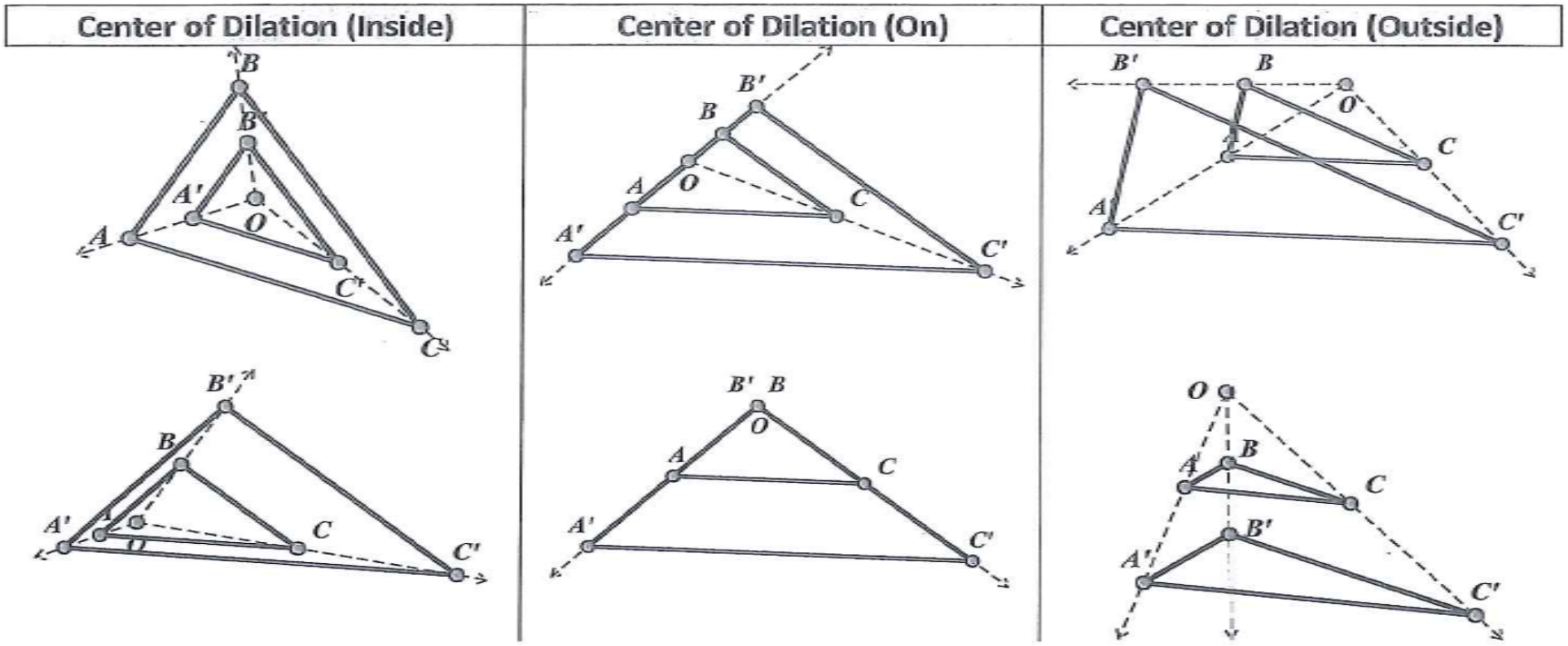
# Using rays to represent and create dilations:




What do you notice?
What do you wonder?

Suppose A'B' = 24.

But here's the fun part.
Dilations have centers. We dilate about a point.
Discuss with a partner what you see



# Are you ready to do some dilations of triangles on graph paper??

**Given triangle ABC, with A (0,0), B (3,1) and C (2,3), dilated with a scale factor of 3 about point O (0,3), find the coordinates of the resulting triangle A'B'C'.**

# A small change:

**Given triangle ABC, with A (0,0),  B (3,1) and C (2,3), dilated with a scale factor of 0.5 about point O (2,2)find the coordinates of the resulting triangle A'B'C'.**

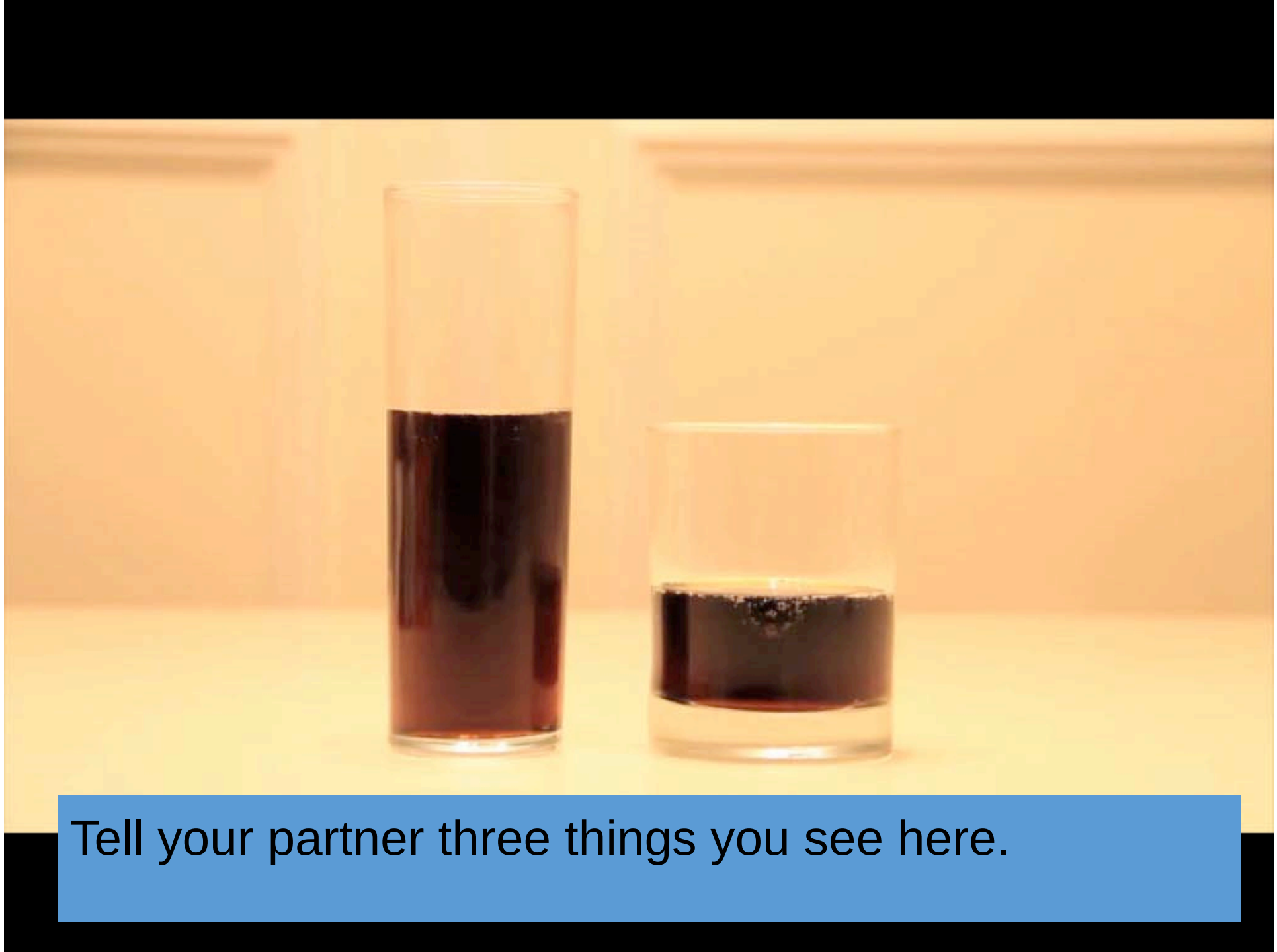
# What did you learn today?

# Exit slip

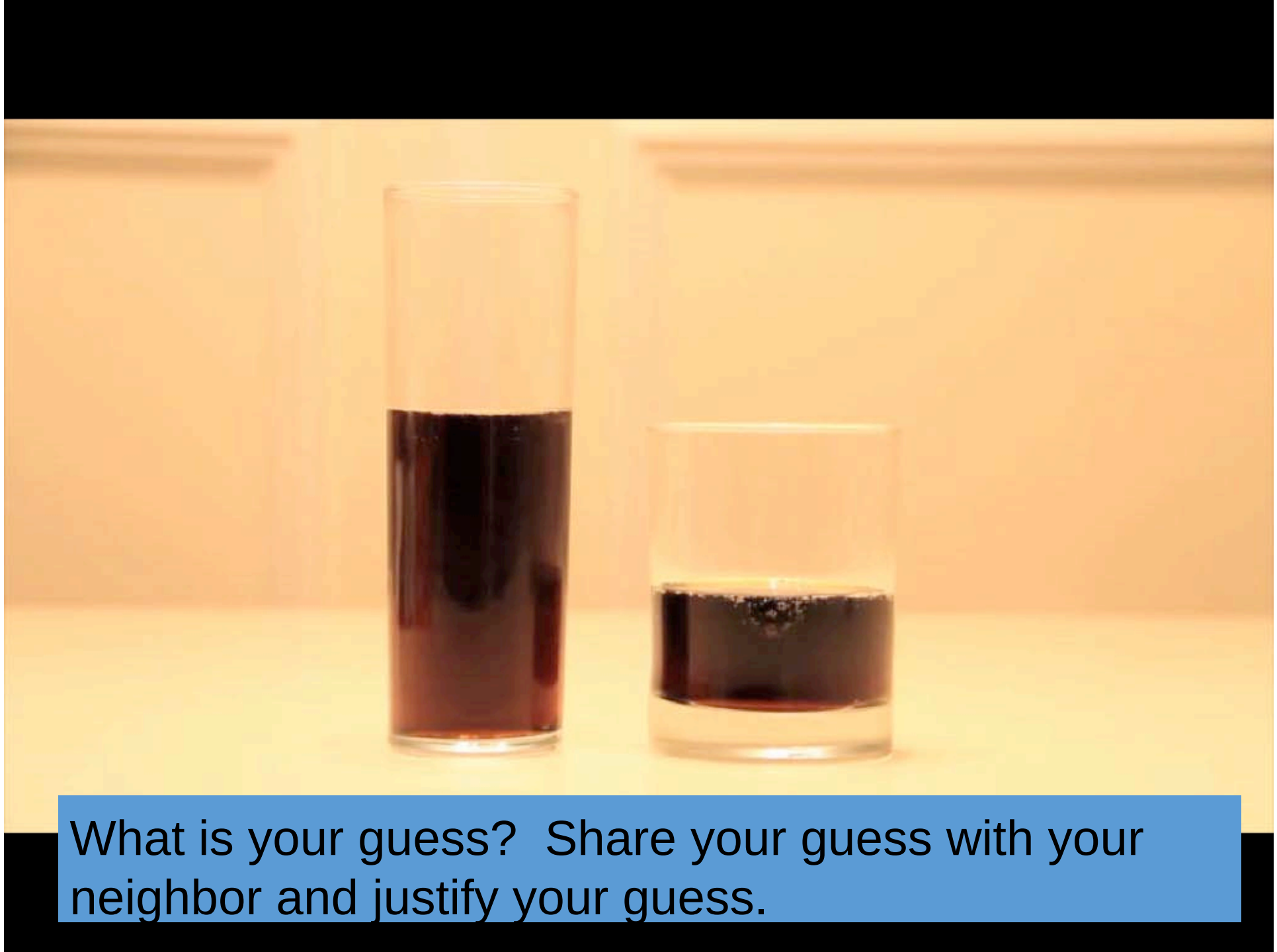
**Draw rectangle ABCD. Now draw rectangle A'B'C'D' that results from a dilation of ABCD with a scale factor of 1/3 about vertex B.**

In closing:

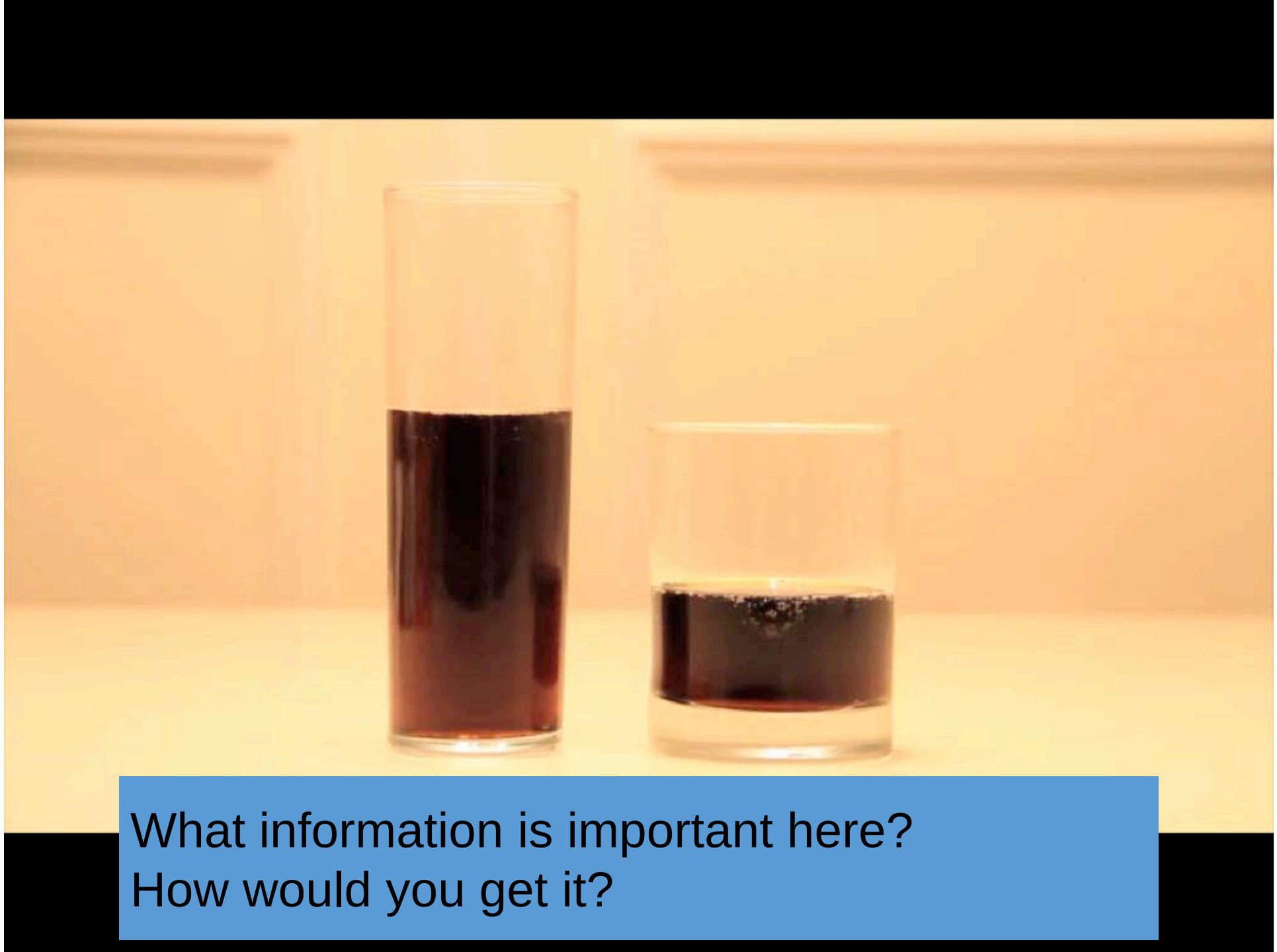
**Turn and tell your neighbor what you learned during this hour.**



Tell your partner three things you see here.



What is your guess?  Share your guess with your neighbor and justify your guess.

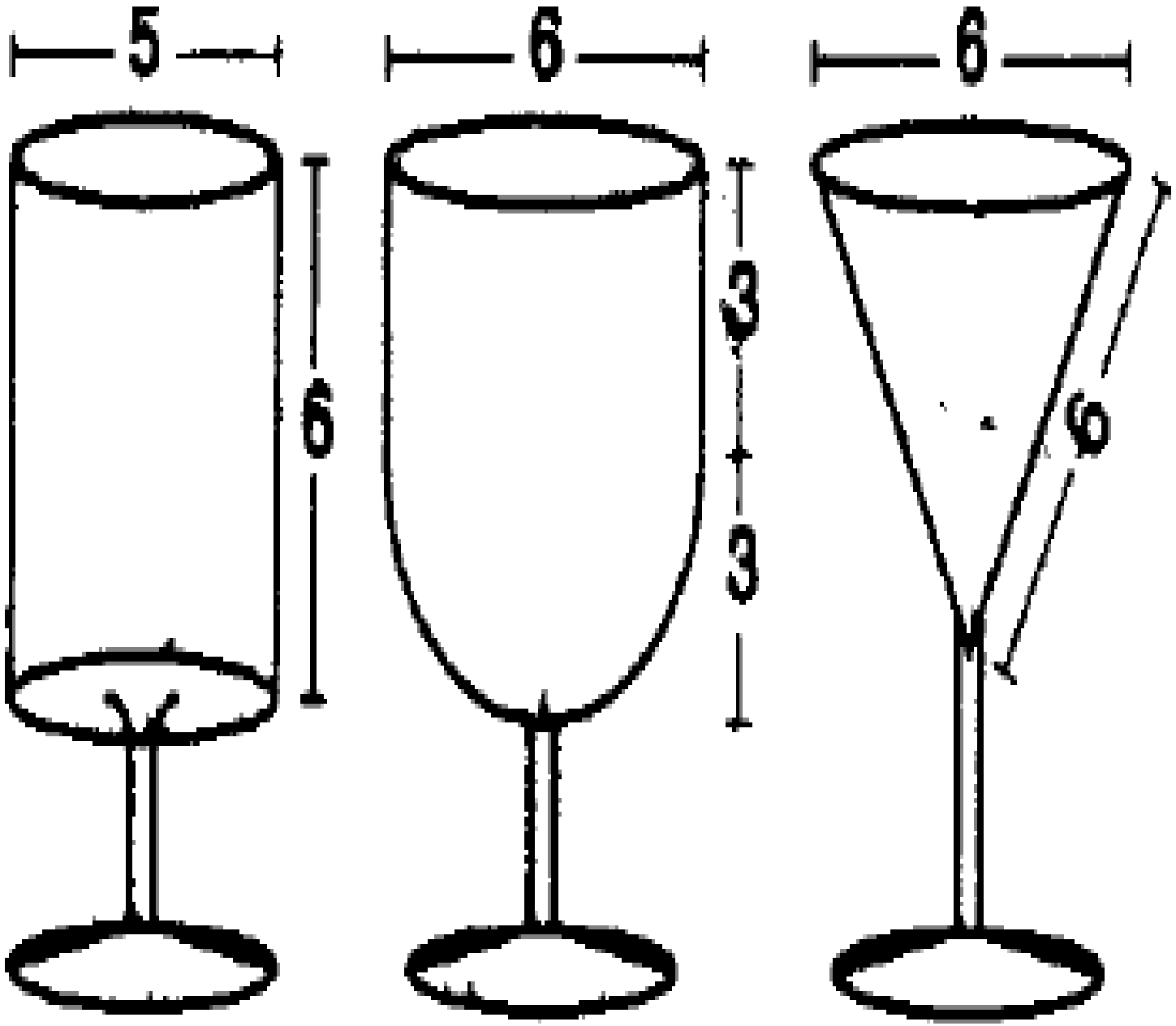


What information is important here?
How would you get it?

# Was anything learned?

- **Now draw two glasses with different diameters and show the heights of equal amounts of liquid. Explain your reasoning.**
- **And on the unit test…..**

Your tu



# So?

- **Order from smallest to largest and justify**
- **What is the height of Glass 3?**
- **What is the volume of each?**
- **If Glass 1 has volume V, express volume of Glasses 2 and 3 in terms of V**
- **When Glass 1 is ½ full, the height of the liquid is 3 cm. What are the heights of the liquid in Glasses 2 and 3 when they are ½ full?**

# A Template for all Lesson Planning

**Mathematics Teaching Practices**

- **Establish mathematics goals to focus learning.**
- **Implement tasks that promote reasoning and problem solving.**
- **Use and connect mathematical representations.**
- **Facilitate meaningful mathematical discourse.**
- **Pose purposeful questions.**
- **Build procedural fluency from conceptual understanding.**
- **Support productive struggle in learning mathematics.**
- **Elicit and use evidence of student thinking.**

# Thank you!

## Go forth and make effective use of the 8 MTPs.